RI : Raw 1 (in kg)
RII : " 2 (in kg)
DI : Drug 1 (in 1000 packs)
DII : Drug 2 (in 1000 packs)

Objective:

max profit

$$\underbrace{(6200\,DI + 6900\,DII)}_{\text{Revenues}} - \underbrace{(100 \cdot RI + 199.9\,RII)}_{\text{Costs}}$$

Constraints:

Agent A: $\underbrace{0.5\,DI + 0.6\,DII}_{\text{Total used}} \le \underbrace{0.01\,RI + 0.02\,RII}_{\text{Total available}}$

$$x_1 + 2.666\,x_2 + 3.5\,x_3 \le 5$$

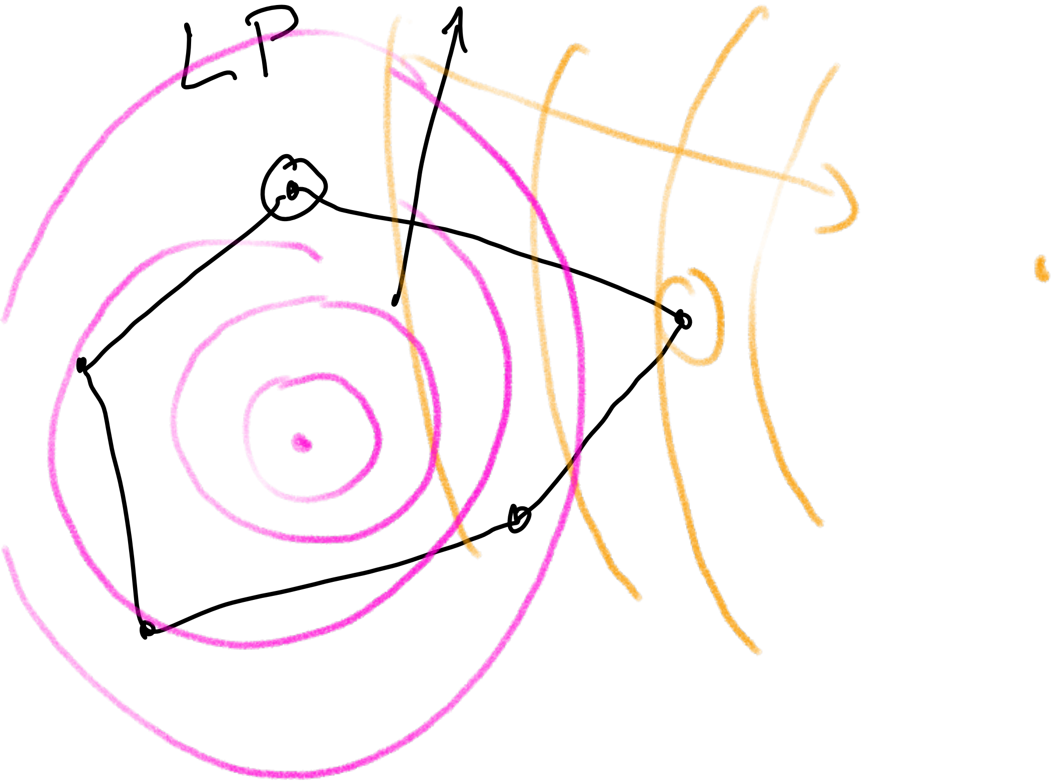
$$\frac{1}{1} \qquad \frac{2666}{1000} \qquad \frac{35}{10}$$

$\frac{p}{q}$ with $q > 100$ then it is uncertain coefficient

$$\tilde{a}_{ij} = (1 + \epsilon\,\xi_{ij})\,a_{ij}$$

$$\xi_{ij} \sim U[-1, 1]$$

LP



$$\begin{bmatrix} \tilde{z}_1 \\ \vdots \\ \vdots \\ \hat{z}_n \end{bmatrix}$$

$(a_1+z_1)x_1 + \cdots (a_n+z_n)x_n \le b$

Robust constraint $\forall z \in Z$

$(a_1+\tilde{z}_1)x_1 \cdots (a_n+\hat{z}_n)x_n \le b$

$\tilde{z}_i \sim F$ $\quad$ $\tilde{z}_i$ are i.i.d

$E_F[\hat{z}_i]=0$, $E_F[\tilde{z}_i^2]=\sigma^2$,

$E_F[(\hat{z}_i^2-\sigma_i^2)^2] = \gamma$ $\quad$ $\}$ Variance $(\hat{z}_i^2)=\gamma$

We are interested in the distribution

of $\|\tilde{z}\|_2 = \sqrt{\tilde{z}^T\tilde{z}}$

$= \sqrt{\sum_{i=1}^n \tilde{z}_i^2} = \sqrt{\tilde{Y}}$

where $\tilde{Y}_n = \sum_{i=1}^n \tilde{z}_i^2$

$\tilde{Y}_n \to N(a, b)$

where $a = E[\tilde{Y}_n] = n\sigma^2$

$b = \text{Variance}\left(\sum_{i=1}^n z_i^2\right) = n\,\text{Variance}(z_i^2)$

$= n\gamma$

With 90% confidence, we have

$$\tilde{Y}_n \in \left[n\sigma^2 - \Phi^{-1}(95\%)\sqrt{n\gamma},\ n\sigma^2 + \Phi^{-1}(95\%)\sqrt{n\gamma} \right]$$

Resume at 11:10 am

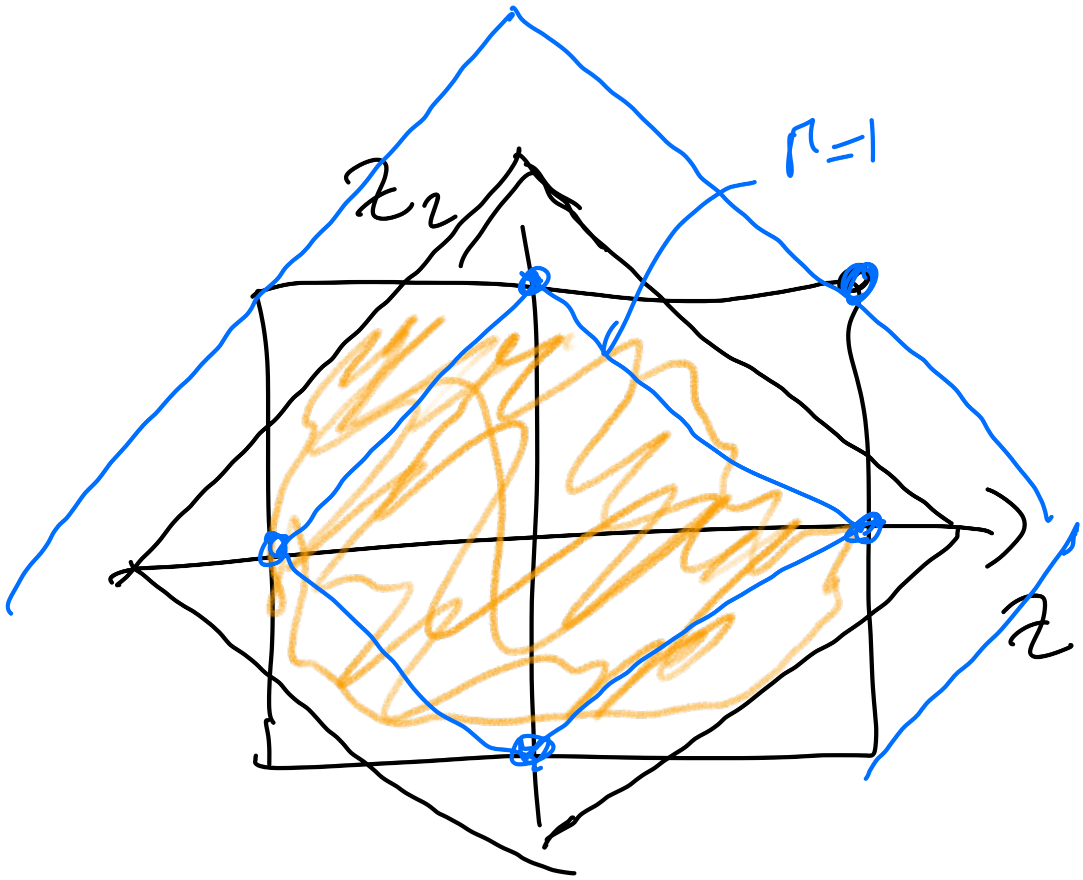
$$\max_{x \in \mathcal{X}} \quad h(x,z)$$

$$\left.\begin{aligned} \max_{x \in \mathcal{X},\, t} \quad & t \\ \text{s.t.} \quad & t \le h(x,z) \end{aligned}\right\} \Rightarrow$$

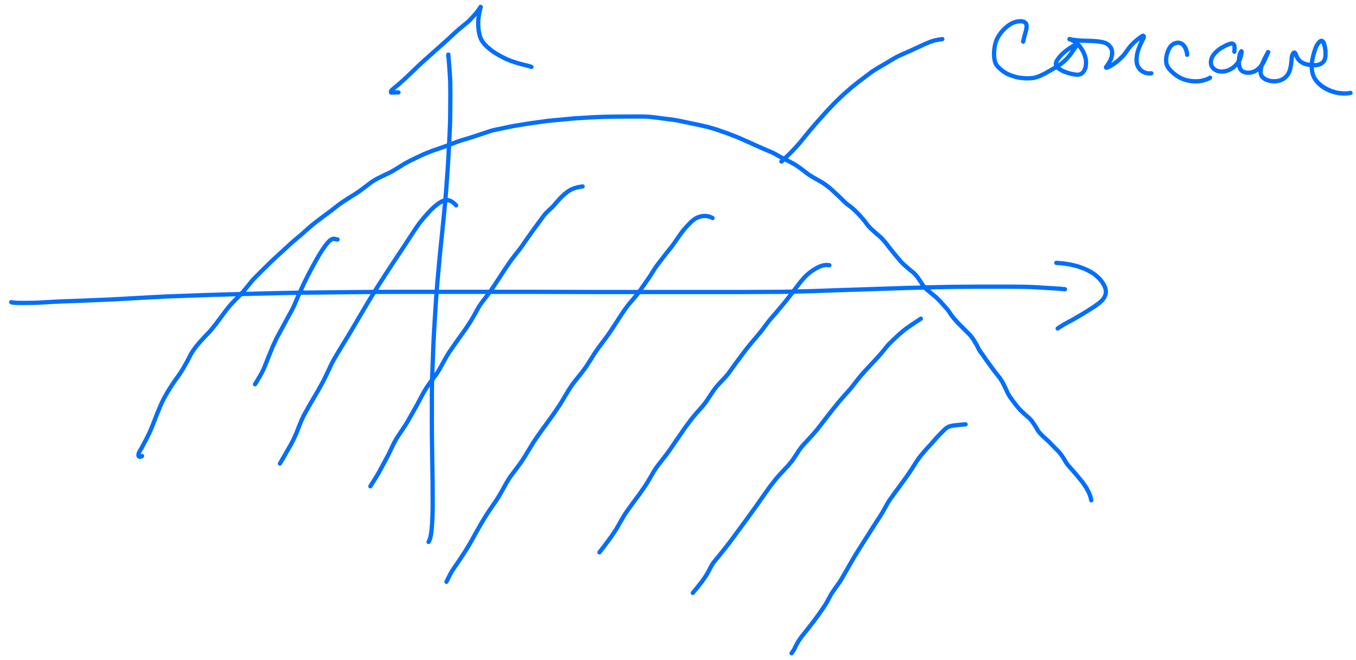
$$\begin{aligned} \max_{x \in \mathcal{X},\, t} \quad & t \\ \text{s.t.} \quad & t \le h(x,z) \;\; \forall z \in \mathcal{Z} \end{aligned}$$

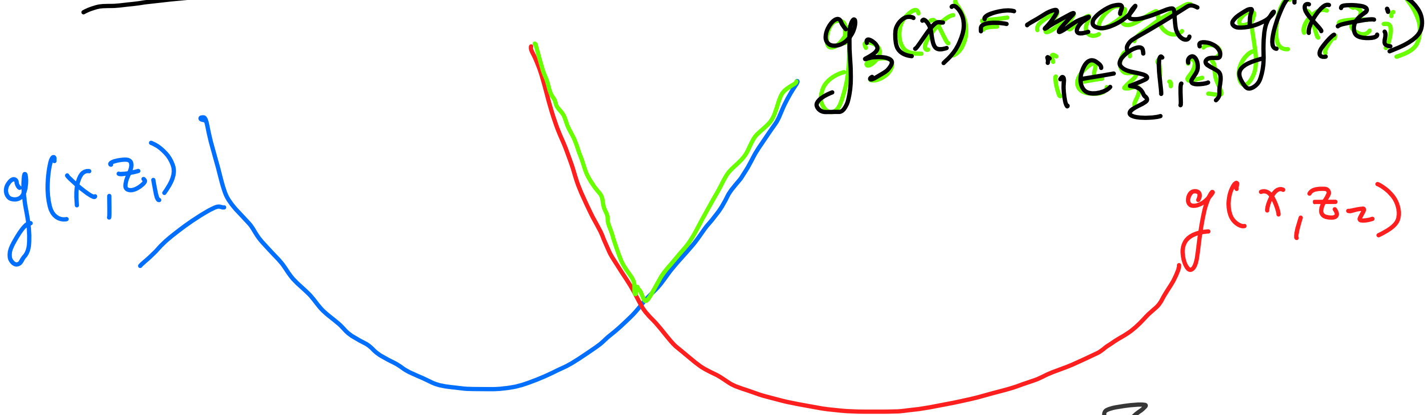
$$\Downarrow$$

$$\max_{x \in \mathcal{X}} \; \min_{z \in \mathcal{Z}} \; h(x,z)$$



x_2
$r=1$
x_1





$g(x,z)$ convex in x $\forall z \in Z$

$h(x) := \sup_{z \in Z} g(x,z)$

$\Rightarrow \forall z, \ \forall \theta, x_1, x_2$, we have that

$$g(\theta x_1 + (1-\theta)x_2, z) \le \theta g(x_1, z) + (1-\theta) g(x_2, z)$$

We can show that $\forall\, \theta, x_1, x_2$

$$h(\theta x_1 + (1-\theta) x_2) = \sup_{z \in Z} g(\theta x_1 + (1-\theta) x_2, z)$$

convexity of g

$$\le \sup_{z \in Z} \theta g(x_1, z) + (1-\theta) g(x_2, z)$$

$$\le \sup_{\substack{z_1 \in Z \\ z_2 \in Z}} \theta g(x_1, z_1) + (1-\theta) g(x_2, z_2)$$

$$= \sup_{z_1 \in Z} \theta g(x_1, z_1) + \sup_{z_2 \in Z} (1-\theta) g(x_2, z_2)$$

$$= \theta \sup_{z_1 \in Z} g(x_1, z_1) + (1-\theta) \sup_{z_2 \in Z} g(x_2, z_2)$$

$$= \theta h(x_1) + (1-\theta) h(x_2)$$

E.g $\quad f(x) = e^{3x+2}$ convex

$$f(x) = e^{x^2}$$

$$= h(g(x))$$

$h(y) = e^y$ convex, increasing

$g(x) = x^2$ convex

$f(x)$ is convex function
by composition rule.